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THREE DIMENSIONAL FLUCTUATING FLOW AND HEAT TRANSFER ALONG A PLATE WITH SUCTION

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Abstract—The flow and heat transfer along a porous plate are investigated when a transverse sinusoidal suction velocity distribution fluctuating with time is applied. Due to this transverse velocity the flow of fluid is three dimensional. For asymptotic flow conditions, wall shear stress and rate of heat transfer are obtained. When frequency parameter $\alpha \to 0$, it is found that the phase lead of the skin friction in the cross-flow direction is $\pi/2$.

NOMENCLATURE

t, time;
$\bar{t}$, dimensionless time, ct;
x, co-ordinate along main flow direction;
y, co-ordinate perpendicular to the plate;
$\bar{y}$, dimensionless co-ordinate perpendicular to the plate, y/l;
z, co-ordinate transverse to the main flow direction;
$\bar{z}$, dimensionless co-ordinate transverse to the main flow direction;
u, velocity component along x;
v, velocity component along y;
w, velocity component along z;
T, fluid temperature;
θ, dimensionless fluid temperature, $\dfrac{T-T_w}{T_\infty - T_w}$;
T, free stream temperature;
T_w, temperature at the plate;
l, wave length of the periodic suction;
V, velocity distribution;
v_0, constant suction velocity;
p, pressure;
ρ, density;
U_∞, free stream velocity;
a, thermal diffusivity;
ν, kinematic viscosity;
Pr, Prandtl number, ν/a;
ε, amplitude of the suction velocity variation;
Re, Reynolds number;
α, frequency parameter, cl^2/ν;
τ_x, skin friction along main flow;
τ_z, skin friction transverse to the main flow;
q_w, local heat transfer at the plate;
Nu, local Nusselt number.

INTRODUCTION

THE PROBLEM of laminar flow control is gaining considerable importance in the fields of aeronautical engineering in view of its application to reduce drag and hence the vehicle power requirements by a substantial amount. The development, since World War II, on this subject has been reported by Lachmann [1]. The transition from laminar to turbulent flow which causes increase of drag coefficient, may be prevented by removing the decelerated particles from the boundary layer through the pores or slits in the wall. The effects of different arrangements and configurations of the suction holes and slits on the drag coefficients have been studied extensively both experimentally and theoretically. Most of the investigators have however, confined themselves to two dimensional flows. There may arise situations where the flow fields may be essentially three dimensional, for example, when variation in the suction velocity distribution is transverse to the potential flow.

Recently Gersten and Gross [2] have studied the effect of a transverse* sinusoidal suction velocity distribution on flow and heat transfer over a plane wall. The suction velocity v_w was assumed in the form $v_w = v_0(1 + \varepsilon \cos \pi z/l)$, $\varepsilon \ll 1$, transverse to the flow direction. Here $v_0 < 0$ for suction, l is the wave length of the periodic suction velocity distribution and ε is the amplitude of the suction velocity variation.

In the present paper we investigate the effect of fluctuating suction velocity distribution on wall shear stress and heat transfer along a porous wall. It is assumed that the intensity of suction velocity also varies with time and is perpendicular to the flow direction. A constant suction velocity at the wall leads to two dimensional flow and any other suction velocity transverse to the flow direction leads to cross flow and hence the flow will be three dimensional over the entire surface. Gersten and Gross [2] have studied the same problem when the suction velocity is independent of time, that is their investigation comes out as a particular case of this work. When the suction velocity becomes independent of time, we have found that Gersten and Gross [2] results are different from those of ours. The correct expressions for main flow field and temperature profile are given. Consequently the expressions for rate of heat transfer and shear stress are also obtained correctly.

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BASIC EQUATIONS

Introduce a coordinate system with the wall lying on x–z plane and y axis perpendicular to it and directed into the incompressible fluid flowing laminarly. We take the suction velocity distribution of the form:

$$v_w(z,t) = v_0\left[1+\varepsilon\cos\left(\frac{\pi z}{l}-ct\right)\right] \tag{1}$$

which consists of a basic steady distribution $v_0 < 0$ with a superimposed weak time varying distribution $v_0\cos[(\pi z/l)-ct]$ of wave length l and frequency c.

We select for investigation the asymptotic flow along the direction of x and hence the velocity and temperature fields will be independent of x, but the flow remains essentially three dimensional due to suction velocity (1).

Denoting velocity components (x, y, z) as (u, v, w) and temperature as T, the equations of continuity Naviers-Stokes and energy reduce to

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{2.1}$$

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\nu\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) \tag{2.2}$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \tag{2.3}$$

$$\frac{\partial w}{\partial t}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right) \tag{2.4}$$

$$\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=a\left(\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right) \tag{2.5}$$

where ν is the kinematic viscosity, ρ is the density, p is the pressure and a is the thermal diffusivity.

The boundary conditions of the problem are

$$y=0:\ u=w=0,\quad v=v_w(z,t)=v_0\left[1+\varepsilon\cos\left(\frac{\pi z}{l}-ct\right)\right],\quad T=T_w; \tag{3.1}$$

$$y\to\infty:\ u=U_\infty,\quad v=v_0,\quad w=0,\quad p=p_\infty,\quad T=T_\infty. \tag{3.2}$$

Method of solution

When the amplitude of the suction velocity variation is small, we assume the solutions of the following form

$$u=u_0+\varepsilon u_1+\varepsilon^2 u_2+\dots \tag{4.1}$$

$$v=V_0+\varepsilon v_1+\varepsilon^2 v_2+\dots \tag{4.2}$$

$$w=w_0+\varepsilon w_1+\varepsilon^2 w_2+\dots \tag{4.3}$$

$$p=p_0+\varepsilon p_1+\varepsilon^2 p_2+\dots \tag{4.4}$$

$$\theta=\theta_0+\varepsilon\theta_1+\varepsilon^2\theta_2+\dots \tag{4.5}$$

where

$$\theta=\frac{T-T_w}{T_\infty-T_w}.$$

In the case $\varepsilon = 0$, we note that u_0, V_0, w_0, p_0 and θ_0 lead to the well known asymptotic solution of two dimensional flow over a plane wall with constant suction and are given as [3]

$$u_0=U_\infty[1-\exp(v_0 y/\nu)];\quad V_0=v_0,\quad w_0=0,\quad p_0=p_\infty\theta_0=1-\exp(v_0 y/a). \tag{5}$$

The series expansion (4) along with (5) are substituted in equations (2) and like powers of ε are equated to get the perturbation equations of various order of ε. For small values of ε it is sufficient to consider the perturbation equations of only $O(\varepsilon)$, which are,

$$\frac{\partial v_1}{\partial y}+\frac{\partial w_1}{\partial z}=0, \tag{6.1}$$

$$\frac{\partial u_1}{\partial t}+v_0\frac{\partial u_1}{\partial y}-\frac{U_\infty v_0}{\nu}\exp(v_0 y/\nu)v_1=\nu\left(\frac{\partial^2 u_1}{\partial y^2}+\frac{\partial^2 u_1}{\partial z^2}\right), \tag{6.2}$$

$$\frac{\partial v_1}{\partial t}+v_0\frac{\partial v_1}{\partial y}=-\frac{1}{\rho}\frac{\partial p_1}{\partial y}+\nu\left(\frac{\partial^2 v_1}{\partial y^2}+\frac{\partial^2 v_1}{\partial z^2}\right), \tag{6.3}$$

$$\frac{\partial w_1}{\partial t}+v_0\frac{\partial w_1}{\partial y}=-\frac{1}{\rho}\frac{\partial p_1}{\partial z}+\nu\left(\frac{\partial^2 w_1}{\partial y^2}+\frac{\partial^2 w_1}{\partial z^2}\right), \tag{6.4}$$

$$\frac{\partial \theta_1}{\partial t}+v_0\frac{\partial \theta_1}{\partial y}-\frac{v_0}{a}\exp(v_0 y/a)v_1=a\left(\frac{\partial^2 \theta_1}{\partial y^2}+\frac{\partial^2 \theta_1}{\partial z^2}\right). \tag{6.5}$$

To solve the above set of linear differential equations it is convenient to adopt complex notations for velocity and temperature profiles. The solutions will be obtained in terms of complex notations, the real part of which will have physical significance. We now write u_1, v_1, w_1, p_1 and θ_1 as

$$u_1(\bar{y},\bar{z},\bar{t})=U_\infty u_{11}(\bar{y})\exp[i(\pi\bar{z}-\bar{t})] \tag{7.1}$$

$$v_1(\bar{y},\bar{z},\bar{t})=\pi v_0 v_{11}(\bar{y})\exp[i(\pi\bar{z}-\bar{t})] \tag{7.2}$$

$$w_1(\bar{y},\bar{z},\bar{t})=iv_0 v'_{11}(\bar{y})\exp[i(\pi\bar{z}-\bar{t})] \tag{7.3}$$

$$p_1(\bar{y},\bar{z},\bar{t})=\rho v_0^2 p_{11}(\bar{y})\exp[i(\pi\bar{z}-\bar{t})] \tag{7.4}$$

$$\theta_1(\bar{y},\bar{z},\bar{t})=\theta_{11}(\bar{y})\exp[i(\pi\bar{z}-\bar{t})] \tag{7.5}$$

where $\bar{z}=z/l$, $\bar{y}=y/l$, $\bar{t}=ct$, and prime denotes differentiation with respect to $\bar{y}$. These forms of velocity profile satisfy the equation of continuity (6.1). Substituting expressions (7) in equations (6), four ordinary differential equations for u_{11}, v_{11}, p_{11} and θ_{11} are obtained:

$$u''_{11}+Reu'_{11}-(\pi^2-i\alpha)u_{11}=-\pi Re^2\exp(-Re\bar{y})v_{11} \tag{8.1}$$

$$v''_{11}+Rev'_{11}-(\pi^2-i\alpha)v_{11}=-\frac{Re}{\pi}p'_{11} \tag{8.2}$$

$$v'''_{11}+Rev''_{11}-(\pi^2-i\alpha)v'_{11}=-\pi Rep_{11} \tag{8.3}$$

$$\theta''_{11}+PrRe\theta'_{11}-(\pi^2-iPr\alpha)\theta_{11}=-\pi Pr^2Re^2\exp(-PrRe\bar{y})v_{11} \tag{8.4}$$

where

$Re=-\dfrac{v_0 l}{\nu}$ is the Reynolds number

$\alpha=\dfrac{cl^2}{\nu}$ is the frequency parameter

and

$Pr=\dfrac{\nu}{a}$ is the Prandtl number.

The boundary conditions according to equations (3), (4) and (7) can be written as

$$y=0:\quad u_{11}=0,\quad v_{11}=\frac{1}{\pi},\quad v'_{11}=0,\quad \theta_{11}=0 \tag{9.1}$$

$$y\to\infty:\quad u_{11}=0,\quad v_{11}=0,\quad v'_{11}=0,\quad \theta_{11}=0,\quad p_{11}=0. \tag{9.2}$$

We observe from equations (8) that the cross flow solutions v_{11}, p_{11}, and w_{11} are independent of main flow component u_{11} and the temperature field θ_{11}, but it influences the main flow and temperature. Therefore we consider first, the equations (8.2) and (8.3) describing cross flow components.

Eliminating v_{11} from equations (8.2) and (8.3) we get equation for p_{11} as

$$p''_{11}-\pi^2 p_{11}=0. \tag{10}$$

The solution of the equation is

$$p_{11}(y)=P_{11}\exp(-\pi\bar{y}). \tag{11}$$

The equation for $v_{11}(\bar{y})$ can now be written as

$$v''_{11}+Rev'_{11}-(\pi^2-i\alpha)v_{11}=ReP_{11}\exp(-\pi\bar{y}). \tag{12}$$

Incorporating the boundary conditions (9), the solution of equation (12) and the value of the constant P_{11} can be easily obtained by usual method. In view of the transformations (7) the complex expressions for $v_1(\bar{y},\bar{z},\bar{t})$ and

$p_1(\bar{y}, \bar{z}, \bar{t})$ can be written as

$$v_1(\bar{y}, \bar{z}, \bar{t}) = \frac{\pi v_0}{\pi-\lambda}\left\{\exp(-\lambda\bar{y}) - \frac{\lambda}{\pi}\exp(-\pi\bar{y})\right\}\exp[i(\pi\bar{z}-\bar{t})], \tag{13}$$

$$p_1(\bar{y}, \bar{z}, \bar{t}) = \frac{\rho v_0^2}{\pi-\lambda}\left\{\lambda - \lambda\left(\frac{i\alpha}{\pi Re}\right)\right\}\exp(-\pi\bar{y})\exp[i(\pi\bar{z}-\bar{t})], \tag{14}$$

$$w_1(\bar{y}, \bar{z}, \bar{t}) = \frac{iv_0\lambda}{\pi-\lambda}\{\exp(-\pi\bar{y}) - \exp(-\lambda\bar{y})\}\exp[i(\pi\bar{z}-\bar{t})], \tag{15}$$

where λ satisfies the equation

$$\lambda^2 - Re\lambda - \pi^2 + i\alpha = 0 \tag{16}$$

or

$$\lambda = A - iB \tag{17}$$

where A and B are given by

$$A = \frac{Re}{2} + \frac{1}{2^{1/2}}\left\{\left[\left(\pi^2 + \frac{Re^2}{4}\right)^2 + \alpha^2\right]^{1/2} + \left(\pi^2 + \frac{Re^2}{4}\right)\right\}^{1/2} \tag{18}$$

$$B = \frac{1}{2^{1/2}}\left\{\left[\left(\pi^2 + \frac{Re^2}{4}\right) + \alpha^2\right] - \left(\pi^2 + \frac{Re^2}{4}\right)\right\}^{1/2}. \tag{19}$$

It is remarkable here that for $\alpha = 0$, the real parts of (13)–(15) are the same as obtained by Gersten and Gross [2]. Using the result of equation (13), we next find the solutions to the equations (8.1) and (8.4), describing main flow u_{11} and temperature θ_{11} in a similar manner. The expressions for u_1 and θ_1 satisfying the boundary conditions (9) are found as

$$u_1 = \frac{U_\infty Re}{\pi-\lambda}\left[\left(\frac{\pi}{2\lambda} - \frac{\lambda Re}{\pi Re + i\alpha}\right)\exp(-\lambda\bar{y}) - \frac{\pi}{2\lambda}\exp[-(Re+\lambda)\bar{y}] + \frac{\lambda Re}{\pi Re + i\alpha}\exp[-(Re+\pi)\bar{y}]\right]\exp[i(\pi\bar{z}-\bar{t})] \tag{20}$$

$$\theta_1(\bar{y}, \bar{z}, \bar{t}) = \frac{-(\sigma Re)^2}{\pi-\lambda}\left[\left\{\frac{\lambda}{Pr(\pi Re + i\alpha)} - \frac{\pi}{Re\lambda(Pr+1) + i\alpha(Pr-1)}\right\}\exp(-\bar{\lambda}\bar{y}) + \frac{\pi}{Re\lambda(Pr+1) + i\alpha(Pr-1)}\exp[-(PrRe+\lambda)\bar{y}] - \frac{\lambda}{Pr(\pi Re + i\alpha)}\exp[-(PrRe+\pi)\bar{y}]\right]\exp[i(\pi\bar{z}-\bar{t})] \tag{21}$$

where $\bar{\lambda}$ is the positive root of the equation

$$\bar{\lambda}^2 - PrRe\bar{\lambda} - \pi^2 + i\alpha Pr = 0 \tag{22}$$

given by

$$\bar{\lambda} = \bar{A} - i\bar{B} \tag{23}$$

and $\bar{A}$ and $\bar{B}$ are given by

$$\bar{A} = \frac{PrRe}{2} + \frac{1}{2^{1/2}}\left\{\left[\left(\pi^2 + \frac{Pr^2Re^2}{4}\right)^2 + Pr^2\alpha^2\right]^{1/2} + \left(\pi^2 + \frac{Pr^2Re^2}{4}\right)\right\}^{1/2} \tag{24}$$

$$\bar{B} = \frac{1}{2^{1/2}}\left\{\left[\left(\pi^2 + \frac{Pr^2Re^2}{4}\right)^2 + Pr^2\alpha^2\right]^{1/2} - \left(\pi^2 + \frac{Pr^2Re^2}{4}\right)\right\}^{1/2}. \tag{25}$$

It is interesting to mention that for $\alpha = 0$, (20), (21) reduce to

$$u_1 = \frac{U_\infty Re}{\pi-\lambda}\left[\left(\frac{\pi}{2\lambda} - \frac{\lambda}{\pi}\right)\exp(-\lambda\bar{y}) - \frac{\pi}{2\lambda}\exp[-(Re+\lambda)\bar{y}] + \frac{\lambda}{\pi}\exp[-(Re+\pi)\bar{y}]\right]\exp(i\pi\bar{z}) \tag{26}$$

$$\theta_1 = \frac{PrRe}{\pi-\lambda}\left\{\left[\frac{\pi Pr}{\lambda(Pr+1)} - \frac{\lambda}{\pi}\right]\exp(-\bar{\lambda}\bar{y}) - \frac{1}{\lambda}\left(\frac{\pi Pr}{Pr+1}\right)\exp[-(PrRe+\lambda)\bar{y}] + \frac{\lambda}{\pi}\exp[-(PrRe+\pi)\bar{y}]\right\}\exp(i\pi\bar{z}). \tag{27}$$

The comparison of expressions (26) and (27) with those obtained by Gersten and Gross [2] reveals that their results for u_1 and θ_1 are different from those of ours because of some calculation mistake in their paper.

RESULTS

The important characteristics of the problem are the wall shear stress and the rate of heat transfer at the wall. The dimensionless shear components along and perpendicular to the main flow directions can now be calculated from the equations (4), (5), (15) and (20) and can be put in the form

$$\tau_x = -\frac{\mu}{\rho v_0 U_\infty}\left(\text{Real}\,\frac{\partial u}{\partial y}\right)_{y=0} = 1+\varepsilon Re_1\cos(\pi z - t + \phi_1) \tag{28}$$

$$\tau_z = -\frac{\mu}{\left(\frac{\mu v_0}{l}\right)}\left(\text{Real}\,\frac{\partial w}{\partial y}\right)_{y=0} = \varepsilon Re_2\cos(\pi z - t + \phi_2) \tag{29}$$

where

$$Re_1 = (C_1^2+D_1^2)^{1/2},\quad Re_2 = (A^2+B^2)^{1/2},\quad \phi_1 = \tan^{-1}\frac{D_1}{C_1}\quad\text{and}\quad \phi_2 = \tan^{-1}\frac{A}{B}$$

and

$$C_1 = \frac{Re}{2}\left(\frac{A}{A^2+B^2} + \frac{1}{\pi BRe-\alpha(\pi-A)}\left\{\frac{2\pi^2 Re[\pi BRe-\alpha(\pi-A)-\alpha Re]}{\pi^2 Re^2+\alpha^2} + \frac{(\pi-A)[\pi BRe+\alpha(\pi-A)}{(\pi-A)^2+B^2}\right\}\right)$$

$$D_1 = \frac{Re}{2}\left(\frac{B}{A^2+B^2} - \frac{1}{\pi BRe-\alpha(\pi-A)}\left\{\frac{2\pi\alpha[\pi BRe-\alpha(\pi-A)-\alpha Re]}{-\pi^2 Re^2+\alpha^2} + \frac{B[\pi BRe+\alpha(\pi-A)]}{(\pi-A)^2+B^2}\right\}\right).$$

The expression for shear stress in the direction of main flow is obtained for $\alpha = 0$ as

$$\tau_x = 1+\varepsilon(1+F_1)\cos\pi z, \tag{30}$$

where

$$F_1 = \frac{\pi(\lambda+\pi)}{2\lambda^2},$$

which is different from that obtained by Gersten and Gross [2]. The magnitude of skin friction in the main flow direction Re_1 is shown in Fig. 1 for various values of α. It is found that for small α, Re_1 increases with α, but for high values of α, Re_1 decreases for small Reynolds number. For higher values of Reynolds number, α has no influence on Re_1. At small Re, Re_2 increases with α while for higher Reynolds number, α has no influence on Re_2. This is shown in Fig. 2. Figure 3 shows that the skin friction in the main flow direction has a phase lead which increases with α. The skin friction perpendicular to main flow direction has a phase lead which increases as α decreases. It is shown in Fig. 4. When $\alpha \to 0$ it is found that $\phi_2 \to \pi/2$.

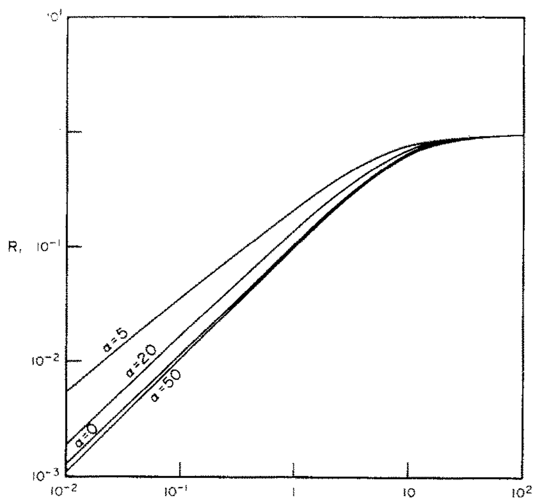

FIG. 1. The amplitude Re_1 vs Re for $\alpha = 0, 5, 20, 50$.

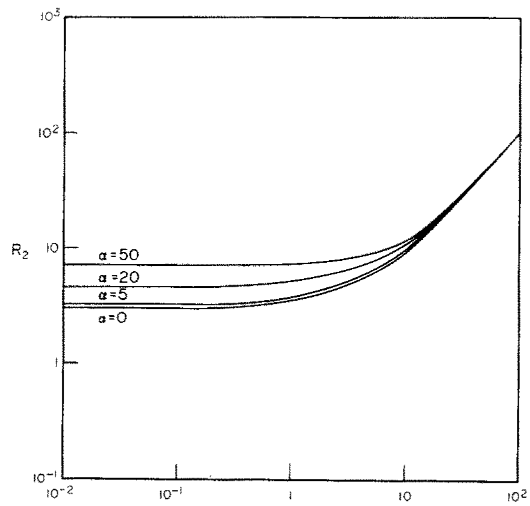

FIG. 2. The amplitude Re_2 vs Re for $\alpha = 0, 5, 20, 50$.

The heat-transfer coefficient from the wall to the fluid may be calculated from the equations (4), (5) and (21) using Fourier's law $q_w = -k(\partial T/\partial y)_{y=0}$ and put in the non-dimensional form

$$Nu = -\frac{q_w}{\rho v_0 C_p(T_w-T_\infty)} = -\frac{k}{\rho v_0 C_p}\left[\text{Real}\,\frac{\partial\theta}{\partial y}\right]_{y=0} = 1+\varepsilon Re_3\cos(\pi z - t + \phi_3) \tag{31}$$

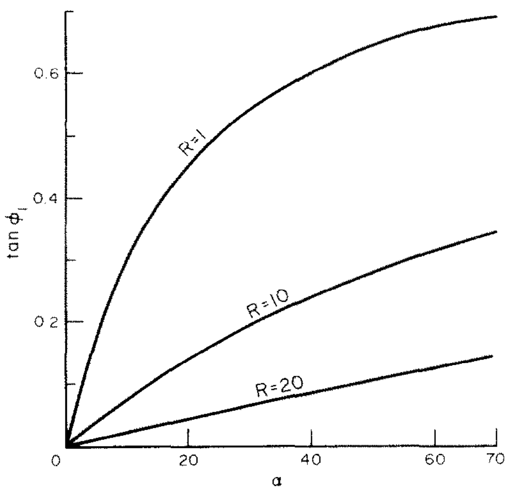

FIG. 3. Tangent of phase shift, $\tan \phi_1$ vs frequency parameter α for $Re = 1, 10, 20$.

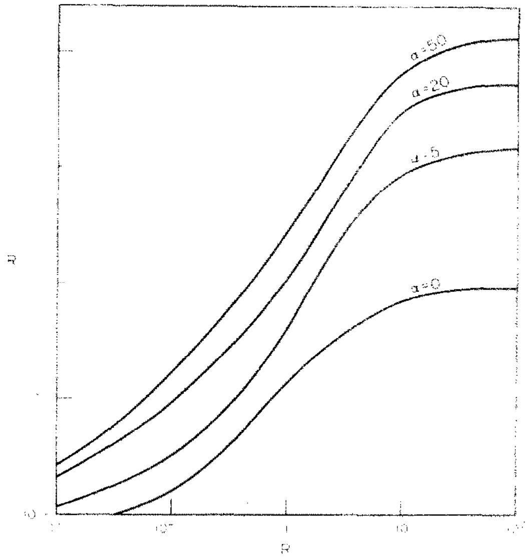

FIG. 5. The amplitude Re_3 in Nusselt number vs Re for $\alpha = 0, 5, 20, 50$.

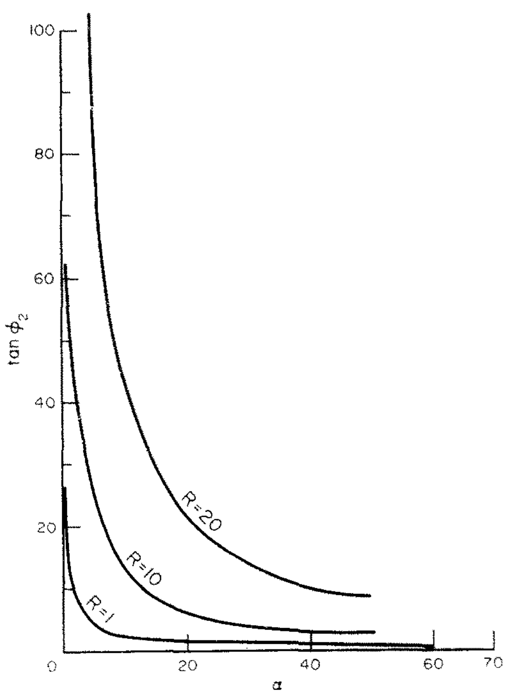

FIG. 4. Tangent of phase shift $\tan \phi_2$ vs frequency parameter α for $Re = 1, 10, 20$.

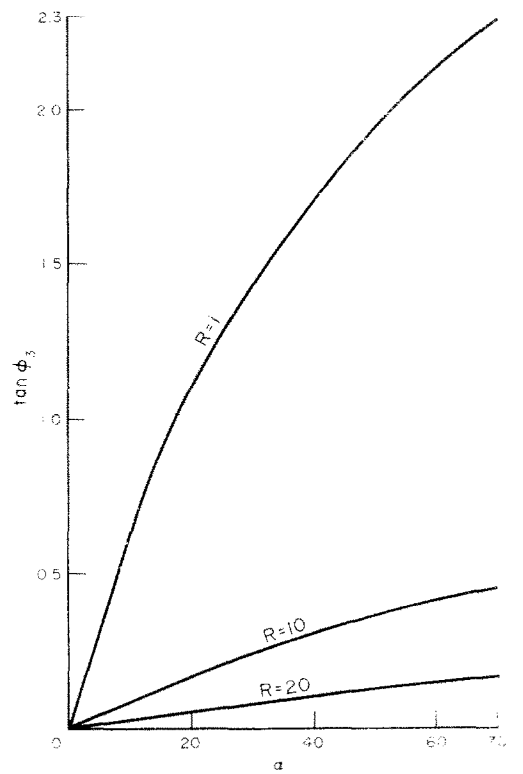

FIG. 6. Tangent of phase shift in Nusselt number, $\tan \phi_3$ vs frequency parameter α for $Re = 1, 10, 20$.

where

$$Re_3 = (C_2^2 + D_2^2)^{1/2}, \quad \phi_3 = \tan^{-1} \frac{D_2}{C_2}$$

$$C_2 = (\bar{X}_1 + \bar{X}_2)(\pi - A) Pr Re + Pr \pi Re^2 \bar{Y}_1 + A Re^2 Pr(Pr+1) \bar{Y}_2$$

$$D_2 = -Pr B Re(\bar{X}_1 + \bar{X}_2) - Pr \alpha Re \bar{Y}_1 + [B Re^2 Pr(Pr+1) - \alpha Re Pr(Pr-1)] \bar{Y}$$

$$\bar{X}_1 = \frac{\pi[B(\bar{A} - Pr Re - \pi)] + \bar{B}[A(\pi - A) - B^2]}{Pr[\pi B Re - \alpha(\pi - A)][(\pi - A)^2 + B^2]}$$

$$\bar{X}_2 = \frac{\pi[B(\pi+PrRe-\bar{A})+\bar{B}(A-\pi)]}{[\alpha(Pr-1)(A-\pi)+\pi BRe(Pr+1)][(\pi-A)^2+B^2]}$$

$$\bar{Y}_1 = \frac{(\pi BRe+\alpha A)(\pi+PrRe-\bar{A})+\bar{A}(\alpha B-\pi ARe)}{Pr[\pi BRe-\alpha(\pi-A)][\pi^2Re^2+\alpha^2]}$$

$$\bar{Y}_2 = \frac{\pi Re(Pr+1)[B(PrRe-\bar{A})+A\bar{B}]-\pi\alpha(Pr-1)(PrRe-\bar{A}+A)}{[\pi BRe(Pr+1)+\alpha(Pr-1)(A-\pi)][A^2Re^2(Pr+1)^2+[\alpha(Pr-1)-BRe(Pr+1)^2]^2\}}.$$

It is found that for $\alpha = 0$, the correct expression for Nusselt number Nu is obtained as

$$Nu = 1+\varepsilon(1-F_3)\cos\pi z \tag{32}$$

where

$$F_3 = \frac{1}{\lambda-\pi}\left\{\left[\frac{\lambda}{\pi}-\frac{\pi Pr}{\lambda(Pr+1)}\right]\bar{\lambda}+\frac{\pi}{\lambda}\left(\frac{Pr^2Re}{Pr+1}\right)+\frac{\pi Pr}{Pr+1}-\lambda\frac{PrRe}{\pi}-\pi\right\} \tag{33}$$

which is different from that obtained in [2]. The amplitude and phase shift in Nusselt number are shown in Fig. 5 and Fig. 6 for $Pr = 0.73$ and for various values of frequency parameter α. It is found that Re_3 increases with α. It means that for higher frequencies rate of heat transfer increases. In Fig. 6 we have plotted the phase shift in Nusselt number against α. It is found that phase lead of Nusselt number increases with α which is more pronounced at small Reynolds number.

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ECOULEMENT FLUCTUANT TRIDIMENSIONNEL ET TRANSFERT THERMIQUE LE LONG D'UNE PLAQUE AVEC ASPIRATION

Résumé—L'écoulement et le transfert thermique le long d'une plaque poreuse sont étudiés quand on applique une distribution fluctuante dans le temps d'une vitesse transversale sinusoïdale. A cause de cette vitesse transversale, l'écoulement est tridimensionnel. On obtient, pour des conditions asymptotiques d'écoulement, la tension de frottement pariétal et le flux thermique. Quand le paramètre de fréquence α tend vers zéro, la phase du frottement pariétal dans la direction transversale est $\pi/2$.

DREIDIMENSIONALE STRÖMUNG UND WÄRMEÜBERGANG LÄNGS EINER PLATTE MIT FLUKTUIERENDER ABSAUGUNG

Zusammenfassung—Die Strömung und der Wärmeübergang längs einer porösen Platte werden untersucht, wenn eine sinusförmige Verteilung der Absauggeschwindigkeit vorliegt, die mit der Zeit fluktuiert. Infolge dieser Quergeschwindigkeit ist die Strömung dreidimensional. Für den Fall asymptotischer Strömungsbedingungen werden Wandscherspannung und Wärmeübergangsgeschwindigkeit bestimmt. Für den Frequenzparameter $\alpha \to 0$ stellt man fest, daß der Phasenvorlauf der Oberflächenreibung quer zur Strömungsrichtung $\pi/2$ ist.

ТРЕХМЕРНОЕ ПУЛЬСИРУЮЩЕЕ ТЕЧЕНИЕ ЖИДКОСТИ И ТЕПЛООБМЕН НА ПЛАСТИНЕ С ОТСОСОМ

Аннотация — Исследуется течение и теплообмен на пористой пластине при флуктуирующем во времени синусоидальном распределении поперечной скорости отсоса. Из-за такого изменения поперечной скорости течение жидкости трехмерно. Для асимптотических условий течения получены значения касательного напряжения на стенке и интенсивности теплообмена. Установлено, что если частотный параметр $\alpha \to 0$, то опережение по фазе поверхностного трения в поперечном направлении составляет $\pi/2$.